Bi-Directional Spheroidal Molecular Communication in Flowing Media

Nadezhda Briantceva
KAUST
Thuwal, Saudi Arabia
nadezhda.briantceva@kaust.edu.sa

Adam Noel
University of Warwick
Coventry, UK
Adam.Noel@warwick.ac.uk

Mohamed-Slim Alouini
KAUST
Thuwal, Saudi Arabia
slim.alouini@kaust.edu.sa

***Abstract*—Molecular communication is an emerging field that explores communication processes at the nano-scale, drawing inspiration from biological systems. As the development of nano-scaled devices advances, the importance of efficient and realistic communication models increases. Thus, this work proposes a novel integration of two-way molecular communication with spheroidal receiver structures, considering the flow dynamics in the communication channel.**

***Index Terms*—Molecular communication, Spheroid, Two-way communication, Flow velocity**

I. Introduction

Molecular communication (MC) is a key technology for nano and micro-scale devices, which enables exchanging information using molecules. Moreover, it facilitates interactions in various settings, including those with spheroidal receivers (RXs) with a cellular porous structure used in our work. In addition to that, our work uniquely integrates the impact of fluid flow within the communication channel (as depicted in Fig. 1), making our model more realistic and setting our study apart from the existing references. It is known that traditional MC has mainly focused on one-way communication, limiting quick responses. At the same time, two-way communication allows devices to receive and reply, making information exchange faster and more efficient. Our study introduces an MC system, enabling bi-directional information exchange between nanodevices. This approach promises to enhance the performance of nano and micro-scale communication systems, making them more efficient and reliable.

II. Molecular Communication Model

A. One-Way Communication Model

Inspired by 3-D cell cultures, specifically spheroids used in organ-on-chip systems, our model has a spheroidal RX structure modeled as a porous medium [1] formed by the groups of absorbing cells. Moreover, we include flow outside of the RXs in our MC channel. Thus, we use the spherical coordinate system (r, θ, φ) to describe the environment geometry. Two diffusive environments with different diffusion coefficients, D^{out} and $D^{\text{in}}_{\text{eff}}$ (outside and inside of the spherical RX) are assumed, thus, the flow continuity condition follows as [1, Eq. (3)]:

$$D^{\text{in}}_{\text{eff}} \frac{\partial c_{\text{in}}(\bar{r}, t)}{\partial r} = D^{\text{out}} \frac{\partial c_{\text{out}}(\bar{r}, t)}{\partial r}, \tag{1}$$

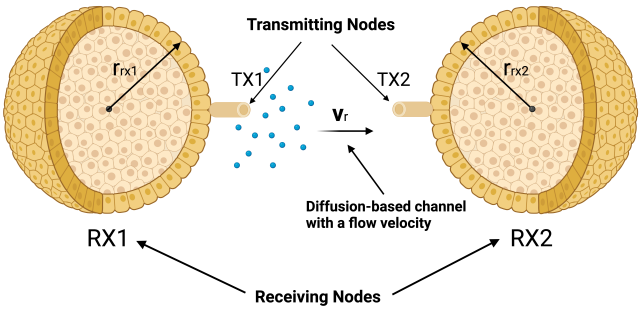

Fig. 1. A two-way diffusion-based molecular communication system with a flow velocity and spheroidal receivers.

with additional boundary condition (BC) as:

$$c_{\text{in}}(\bar{r}, t) = p^{\text{in}} c_{\text{out}}(\bar{r}, t), \tag{2}$$

where $\bar{r} \in \partial\Omega, \partial\Omega$ is the boundary region of the spheroid, p^{in} is a porosity constant of the spherical environment, and c_{in} and c_{out} denote the concentration function inside and outside the spheroid, respectively. To represent the release of the IMs from the transmitter (TX), the impulsive point source is introduced as: $S(\bar{r}, t, \bar{r}_0, t_0) = \frac{\delta(r-r_0)\delta(\theta-\theta_0)\delta(\varphi-\varphi_0)\delta(t-t_0)}{r^2 \sin\theta} \text{s}^{-1}\ \text{m}^{-3}$. Therefore, the molecular advection-diffusion outside of the RX spheroid can be described by the partial differential equation (PDE):

$$\begin{aligned} &D^{\text{out}} \nabla^2 c_{\text{out}}(\bar{r}, t \mid \bar{r}_o, t_0) - V_r \nabla c_{\text{out}}(\bar{r}, t \mid \bar{r}_0, t_0) \\ &+ S(\bar{r}, t, \bar{r}_0, t_0) = \frac{\partial c_{\text{out}}(\bar{r}, t \mid \bar{r}_0, t_0)}{\partial t}, \end{aligned} \tag{3}$$

where $c_{\text{out}}(\bar{r}, t \mid \bar{r}_0, t_0)$ denotes the molecule concentration at point $\bar{r}$ and time t. Additionally, V_r represents both the speed and direction of the uniform flow in the system.

Moreover, the concentration inside of the spheroidal RX $c_{\text{in}}(\bar{r}, t \mid \bar{r}_0, t_0)$ can be found by solving the following PDE:

$$D^{\text{in}}_{\text{eff}} \nabla^2 c_{\text{in}}(\bar{r}, t \mid \bar{r}_o, t_0) - k_d c_{\text{in}}(\bar{r}, t \mid \bar{r}_0, t_0) = \frac{\partial c_{\text{in}}(\bar{r}, t \mid \bar{r}_0, t_0)}{\partial t}, \tag{4}$$

where k_d is the degradation constant, which tells us how fast the IMs are absorbed by the cells inside of the RX.

By writing both equations (3)-(4) along with the BCs in a Fourier domain, their solutions can be presented as [2, Eq. (7)]:

$$C^l\left(r,\theta,\varphi,\omega \mid \bar{r}_{\mathrm{tx}}\right)=\sum_{n=0}^{\infty}\sum_{m=0}^{n} H_{mn}R_n^l(r,\omega)\cos\left(m\left(\varphi-\varphi_{\mathrm{tx}}\right)\right) \times P_{nm}(\cos\theta), \quad l\in\{\mathrm{in},\mathrm{out}\}, \tag{5}$$

where $R_n^l(r,\omega)$ is the radial function for the region $l \in \{\mathrm{in},\mathrm{out}\}$, which for our problem is:

$$R_n^{\mathrm{in}}(r,\omega)=A_n j_n(pr), r<r_{\mathrm{rx1}}$$
$$R_n^{\mathrm{out}}(r,\omega)=\begin{cases}\sqrt{2r(D^{\mathrm{out}}-V_r)}^{k_1}B_nJ_n\left(k_2,\sqrt{-k_1}pr\right)\\ +C_nY_n\left(k_2,\sqrt{-k_1}pr\right), r_{\mathrm{rx1}}<r<r_{\mathrm{tx1}};\\ G_nH_n^1(pr), r>r_{\mathrm{tx1}},\end{cases} \tag{6}$$

where $k_1=\frac{D^{\mathrm{out}}}{(V_r-D^{\mathrm{out}})}$, $k_2=\frac{\sqrt{(D^{\mathrm{out}}+2D^{\mathrm{out}}n)^2+2D^{\mathrm{out}}V_r-4D^{\mathrm{out}}n(1+n)V_r+V_r^2}}{2(D^{\mathrm{out}}-V_r)}$, $J_n(\cdot)$, $Y_n(\cdot)$ are the first and second types of the Bessel function, respectively, $H_n^1(\cdot)$ is a Hankel Bessel function of the first kind, and $p=\sqrt{-\frac{iw}{D^{\mathrm{out}}}}$.

Thus, by finding the coefficients A_n, B_n, C_n and G_n [3], we can get the desired expressions $c_{\mathrm{in}}(\bar{r},t)$ and $c_{\mathrm{out}}(\bar{r},t)$ by performing the inverse Fourier transform of (5). Therefore, the arrival time distribution for RX1 after the TX1 emits the IMs, which we denote $A_t^{\mathrm{tx1}}(t)$ can be derived as in [3] and used to get a fraction of molecules hitting the RX1 by time t as follows:

$$F_h^{\mathrm{tx1}}(t)=\int_0^t A_t^{\mathrm{tx1}}(t')\,dt'. \tag{7}$$

Now, we focus on establishing a two-way (bi-directional) communication system using a single type of IM.

B. Two-Way Communication Model

We consider a 3-D environment with two point sources, TXs, and two spherical RXs with a porous structure. Fig. 1 demonstrates the model of the system. Each TX emits molecules governed by (3)-(4), correspondingly. In our study, we plan to derive a channel model for a two-way MC system focusing here on IMs emitted from TX1 and their trajectories [4]. Thus, the model, illustrated in Fig. 2, considers three potential diffusion paths. Path 2, leading to RX2, is key to our analysis, while RX1 is assumed to be a nontarget destination. Superposition can be used to obtain the whole TX2 case, so it will not be considered in the derivation.

First, we calculate the hitting probability at RX2, $P_{h,2}^{\mathrm{tx1}}(t)$, by integrating the instantaneous hitting probability density, $p_{h,2}^{\mathrm{tx1}}(t')$, over time. This density is derived by subtracting the probability of IMs following an alternative path (Path 3) from the total hitting probability for RX2, which is the same as for the one-way communication scenario in (7). For IMs undergoing Path 3, we observe that they initially hit RX1 at a point $\epsilon_{1,\mathrm{tx1}}$ at time τ_ϵ. Considering this, we

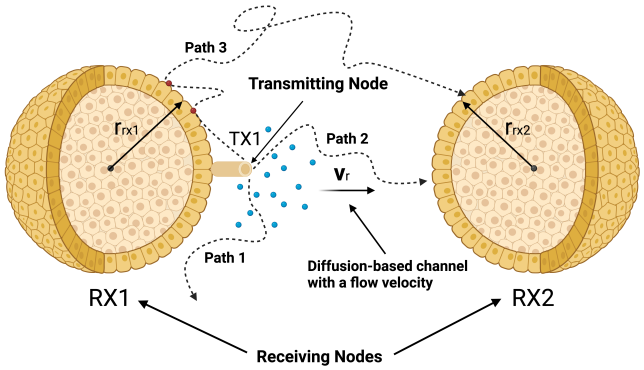

Fig. 2. Three possible paths for the information molecules to follow after emission from TX1.

modify the instantaneous hitting probability density for RX2 as $p_{h,2}^{\mathrm{tx1}}(t')=f_{h,2}^{\mathrm{tx1}}(t')-\int_0^{t'}p_{h,1}^{\mathrm{tx1}}(\tau_\epsilon)f_{h,2}^{s_1^\ell}(t'-\tau_\epsilon)\,d\tau_\epsilon$, where $f_{h,2}^{\mathrm{tx1}}(t')$ represents the probability density for the combined Path 2 and Path 3, and $p_{h,1}^{\mathrm{tx1}}(\tau_\epsilon)$ is the analogous density for molecules initially hitting RX1.

The methodology sets ϵ_i^ℓ as constants to simplify the integration and obtain approximate forms for $P_{h,2}^{\mathrm{tx1}}(t)$ and $P_{h,1}^{\mathrm{tx1}}(t)$, the cumulative hitting probabilities for RX2 and RX1, respectively. After integrating the instantaneous densities, we end up with the following derivations:

$$\begin{aligned}P_{h,2}^{\mathrm{tx1}}(t)&\approx F_{h,2}^{\mathrm{tx1}}(t)-f_{h,2}^{\epsilon_1^\ell}(t)*P_{h,1}^{\mathrm{tx1}}(t)\\ P_{h,1}^{\mathrm{tx1}}(t)&\approx F_{h,1}^{\mathrm{tx1}}(t)-f_{h,1}^{\epsilon_2^\ell}(t)*P_{h,2}^{\mathrm{tx1}}(t).\end{aligned} \tag{8}$$

The evaluation of these channel model approximations is the focus of our ongoing research.

III. Conclusion and Further Plans

This ongoing study contributes to molecular communication, mainly focusing on systems involving larger-scale spheroidal structures, such as those in the micron range. We combine bi-directional communication with spherical, porous receivers and flow dynamics to create a more effective and realistic communication model. In the future, we plan to finalize the derivations for the formulated system and compare the results with simulations. Moreover, we will include and analyze techniques for self-interference cancellation.

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